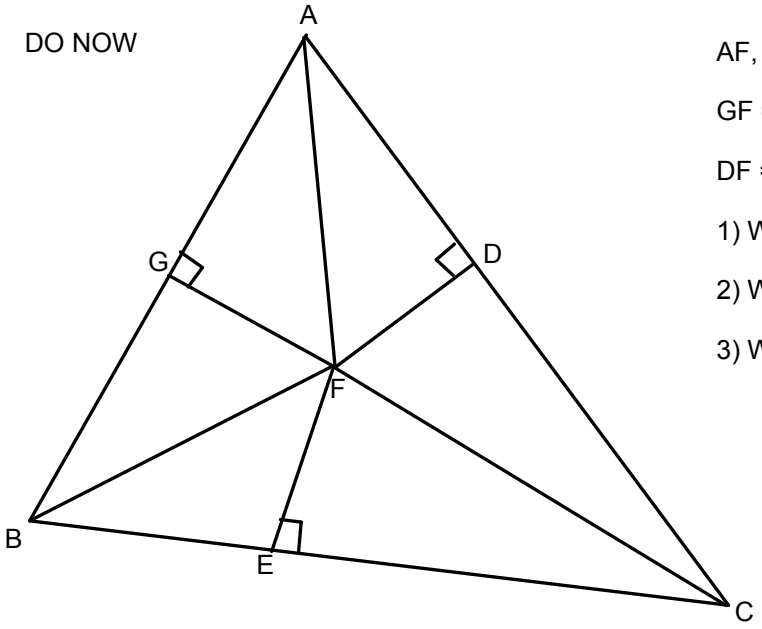
Day 43

DO NOW



AF, CF, and BF are angle bisectors.

GF = 2x + 12

DF = x + 20

1) What is point F called?

2) What is the value of x?

3) What is the length of EF?

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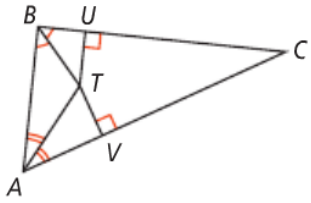
2. In the figure at the right, $TV = 3x - 12$ and $TU = 5x - 24$. What is the value of x?

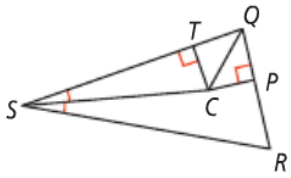


Do you UNDERSTAND?

3. Vocabulary A triangle's circumcenter is outside the triangle. What type of triangle is it?

4. Reasoning You want to find the circumcenter of a triangle. Why do you only need to find the intersection of two of the triangle's perpendicular bisectors, instead of all three?

5. Error Analysis Your friend sees the triangle at the right and concludes that $CT = CP$. What is the error in your friend's reasoning?



6. Compare and Contrast How are the circumcenter and incenter of a triangle alike? How are they different?

2. 6

3. obtuse △

4. Since the three ⊥ bisectors of a △ are concurrent, the third ⊥ bisector goes through the pt. of intersection of the other two ⊥ bisectors.

5. Answers may vary. Sample: The diagram does not show that $\overline{QC}$ bisects $\angle SQR$, so you cannot conclude that point C is equidistant from the sides of $\angle SQR$.

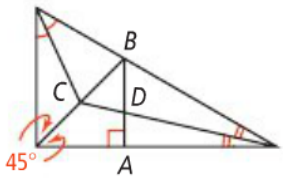
6. Each one is a point of concurrency of bisectors of parts of a △, each is equidistant from three parts of the △, and each is the center of a ⊙ that contains three points of the △. The circumcenter is equidistant from three points, while the incenter is equidistant from three segments. The △ is inside the ⊙ centered at the circumcenter and outside the ⊙ centered at the incenter.

Day 43

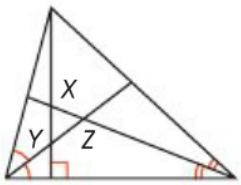
homework

Name the point of concurrency of the angle bisectors.

15.

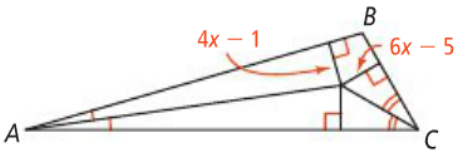


16.

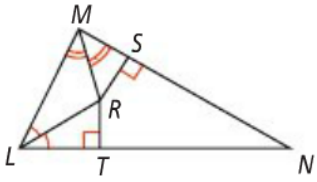


Find the value of x.

17.



18. $RS = 4(x - 3) + 6$ and $RT = 5(2x - 6)$.



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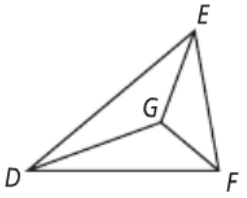
22. Algebra In the diagram at the right, G is the incenter of $\triangle DEF$, $m\angle DEF = 60$, and $m\angle EFD = 2 \cdot m\angle EDF$. What are $m\angle DGE$, $m\angle DGF$, and $m\angle EGF$?



Reasoning **Determine whether each statement is *true* or *false*. If the statement is false, give a counterexample.**

26. The incenter of a triangle is equidistant from all three vertices.

27. The incenter of a triangle always lies inside the triangle.

28. You can circumscribe a circle about any three points in a plane.

29. If point C is the circumcenter of $\triangle PQR$ and the circumcenter of $\triangle PQS$, then R and S must be the same point.

Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.

31. It is possible to find a point equidistant from three parallel lines in a plane.

32. The circles inscribed in and circumscribed about an isosceles triangle have the same center.

33. Which of the following statements is *false*?

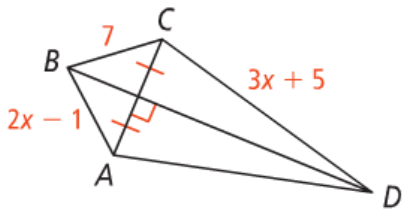
(A) The bisectors of the angles of a triangle are concurrent.

(B) The midsegments of a triangle are concurrent.

(C) The perpendicular bisectors of the sides of a triangle are concurrent.

(D) Four lines intersecting in one point are concurrent.

Use the figure at the right for Exercises 37 and 38.



37. Find the value of x.

38. Find the length of $\overline{AD}$.

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15. C

16. Z

17. 2

18. 4

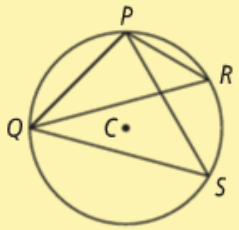
22. $m\angle DGE = 130$, $m\angle DGF = 120$, $m\angle EGF = 110$

26. False; the circumcenter is equidistant from the vertices, and the incenter and circumcenter are different (unless the $\triangle$ is an equilateral $\triangle$).

27. true

28. False; if the points are collinear, then the $\perp$ bisectors of the segments determined by the points will be $\parallel$. Since the $\perp$ bisectors are $\parallel$, they will not intersect, so there is no point that is equidistant from all 3 points.

29.



As the diagram shows, circle C is circumscribed about both $\triangle PQR$ and $\triangle PQS$, so points R and S do not have to coincide.

31. Never
32. sometimes
33. B

37. 4

38. 17

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Day 43

Bisectors in Triangles

Objective To identify properties of perpendicular bisectors and angle bisectors

When three or more lines intersect at one point, they are **concurrent.** The point at which they intersect is the **point of concurrency.**

Essential Understanding For any triangle, certain sets of lines are always concurrent. Two of these sets of lines are the perpendicular bisectors of the triangle's three sides and the bisectors of the triangle's three angles.

Day 43

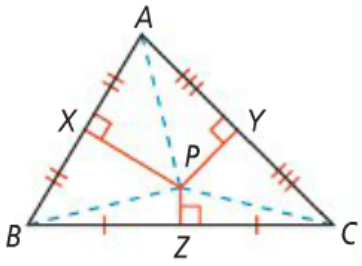
take note

Theorem 5-6 Concurrency of Perpendicular Bisectors Theorem

Theorem

The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices.

Diagram



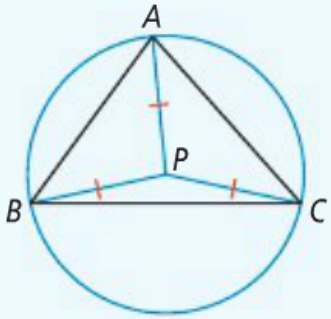
Symbols

Perpendicular bisectors $\overline{PX}$, $\overline{PY}$, and $\overline{PZ}$ are concurrent at P.

$PA = PB = PC$

The point of concurrency of the perpendicular bisectors of a triangle is called the **circumcenter of the triangle.**

Since the circumcenter is equidistant from the vertices, you can use the circumcenter as the center of the circle that contains each vertex of the triangle. You say the circle is **circumscribed about** the triangle.



Day 43

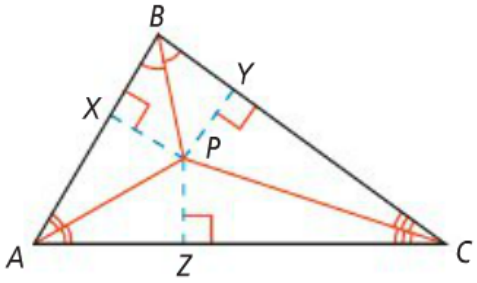
take note

Theorem 5-7 Concurrency of Angle Bisectors Theorem

Theorem

The bisectors of the angles of a triangle are concurrent at a point equidistant from the sides of the triangle.

Diagram



Symbols

Angle bisectors $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are concurrent at P.

$PX = PY = PZ$

Day 43

Medians and Altitudes

Objective To identify properties of medians and altitudes of a triangle

Day 43

Activity #1

1) Find the midpoint of AC and label it D
2) Find the midpoint of AB and label it E
3) Construct a line that goes through BD
4) Construct a line that goes through CE
5) Point F is now the intersection of BD and CE, label it
6) Construct a line now going through A and F so that it passes through side BC. Where the line intersects BC, label it G. Is point G the midpoint of BC? Measure it to show me that it is or is not.
7) Now it's time to measure the following lengths:
(hint, you do not have to measure the last column with a ruler to get the lengths)

AG = AF = FG =

BD = BF = FD =

CE = CF = FE =

8) Now let's do some calculations:

AG/AF = AG/FG = AF/FG =

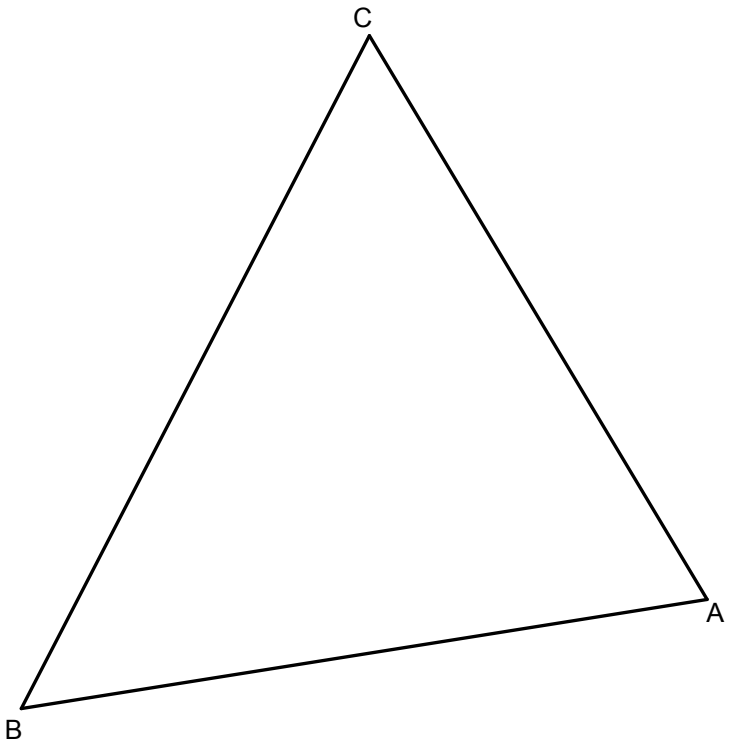
BD/BF = BD/FD = BF/FD =

CE/CF = CE/FE = CF/FE =

9) What's the relationships that you see happening between the segments created? Explain in detail.

Day 43

ACTIVITY #1



Day 43

ACTIVITY #2

1) Find the midpoint of AC and label it D
2) Find the midpoint of AB and label it E
3) Construct a line that goes through BD
4) Construct a line that goes through CE
5) Point F is now the intersection of BD and CE, label it
6) Construct a line now going through A and F so that it passes through side BC. Where the line intersects BC, label it G. Is point G the midpoint of BC? Measure it to show me that it is or is not.
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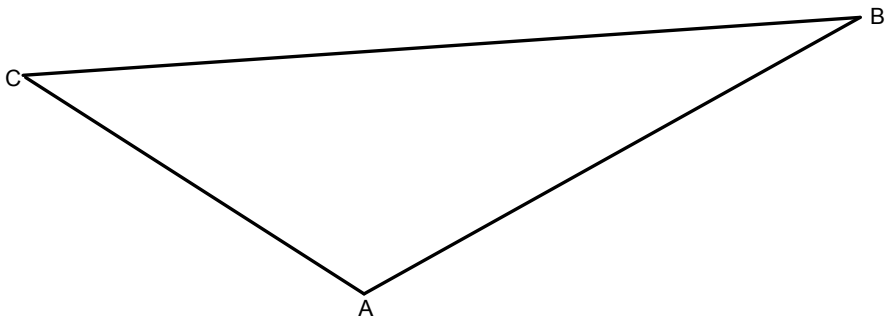
9) What's the relationships that you see happening between the segments created? Explain in detail.

Day 43

ACTIVITY #2



Day 43

ACTIVITY #3

1) Find the midpoint of AC and label it D
2) Find the midpoint of AB and label it E
3) Construct a line that goes through BD
4) Construct a line that goes through CE
5) Point F is now the intersection of BD and CE, label it
6) Construct a line now going through A and F so that it passes through side BC. Where the line intersects BC, label it G. Is point G the midpoint of BC? Measure it to show me that it is or is not.
7) Now it's time to measure the following lengths:
(hint, you do not have to measure the last column with a ruler to get the lengths)

AG = AF = FG =

BD = BF = FD =

CE = CF = FE =

8) Now let's do some calculations:

AG/AF = AG/FG = AF/FG =

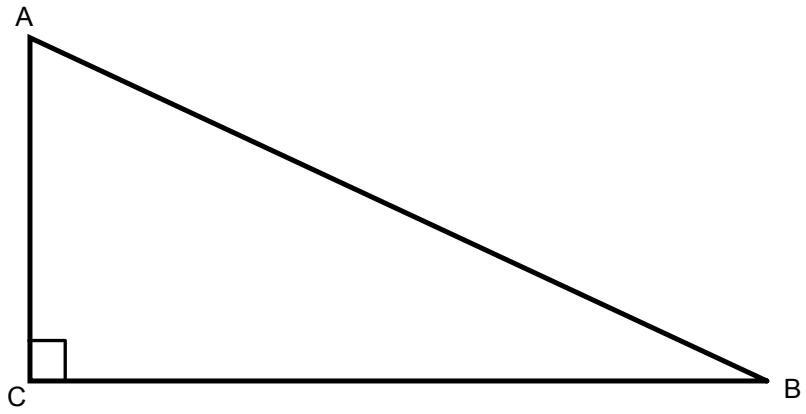
BD/BF = BD/FD = BF/FD =

CE/CF = CE/FE = CF/FE =

9) What's the relationships that you see happening between the segments created? Explain in detail

Day 43

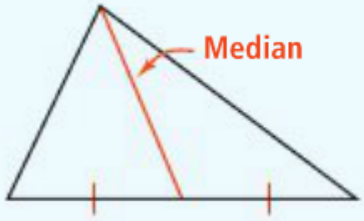
ACTIVITY #3



Day 43

Essential Understanding A triangle's three medians are always concurrent.

A **median of a triangle** is a segment whose endpoints are a vertex and the midpoint of the opposite side.

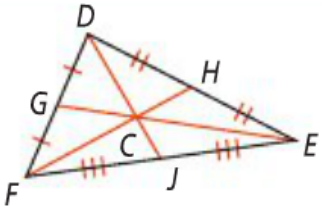


take note

Theorem 5-8 Concurrency of Medians Theorem

The medians of a triangle are concurrent at a point that is two thirds the distance from each vertex to the midpoint of the opposite side.

$DC = \frac{2}{3}DJ$ $EC = \frac{2}{3}EG$ $FC = \frac{2}{3}FH$



In a triangle, the point of concurrency of the medians is the **centroid of the triangle.** The point is also called the *center of gravity* of a triangle because it is the point where a triangular shape will balance. For any triangle, the centroid is always inside the triangle.

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Definition

1) point of concurrency of the medians of a triangle

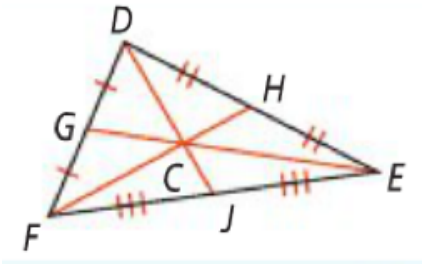
2) center of gravity of a triangle

Properties (see diagram)

GC = (1/3)GE
CE = (2/3)GE

GC = (1/2)CE
CE = 2(GC)

What does it look like?



Point C is the centroid

Centroid

How is it created?

Medians

Medians - a segment that connects a vertex to a midpoint

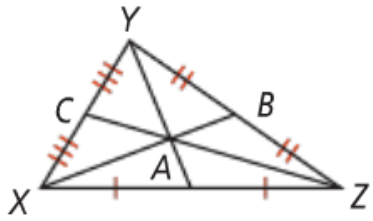
Where is it located?

Acute - inside
Obtuse - inside
Right - inside

Day 43

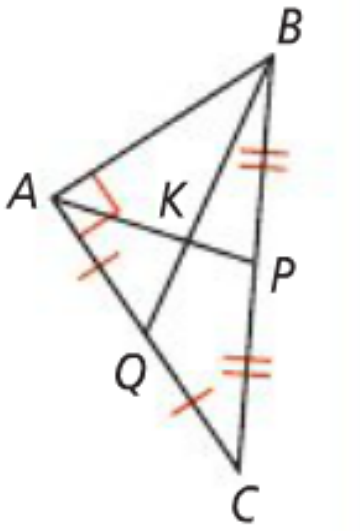
Finding the Length of a Median

In the diagram at the right, $XA = 8$. What is the length of $\overline{XB}$?



2. If $AP = 18$, what is KP?

3. If $BK = 15$, what is KQ?



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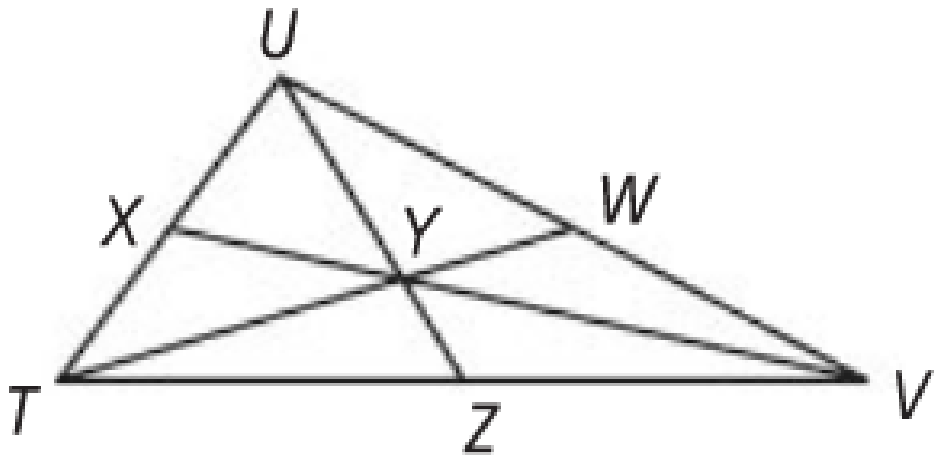
In $\triangle TUV$, Y is the centroid.

8. If $YW = 9$, find TY and TW.

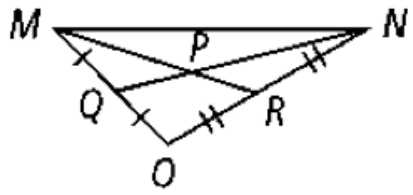
9. If $YU = 9$, find ZY and ZU.

10. If $VX = 9$, find VY and YX.



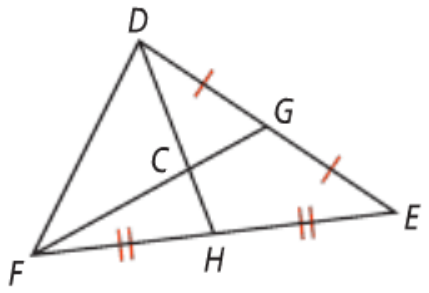
Day 43

P is the centroid of λMNO. $MP = 14x + 8y$. Write expressions to represent PR and MR.

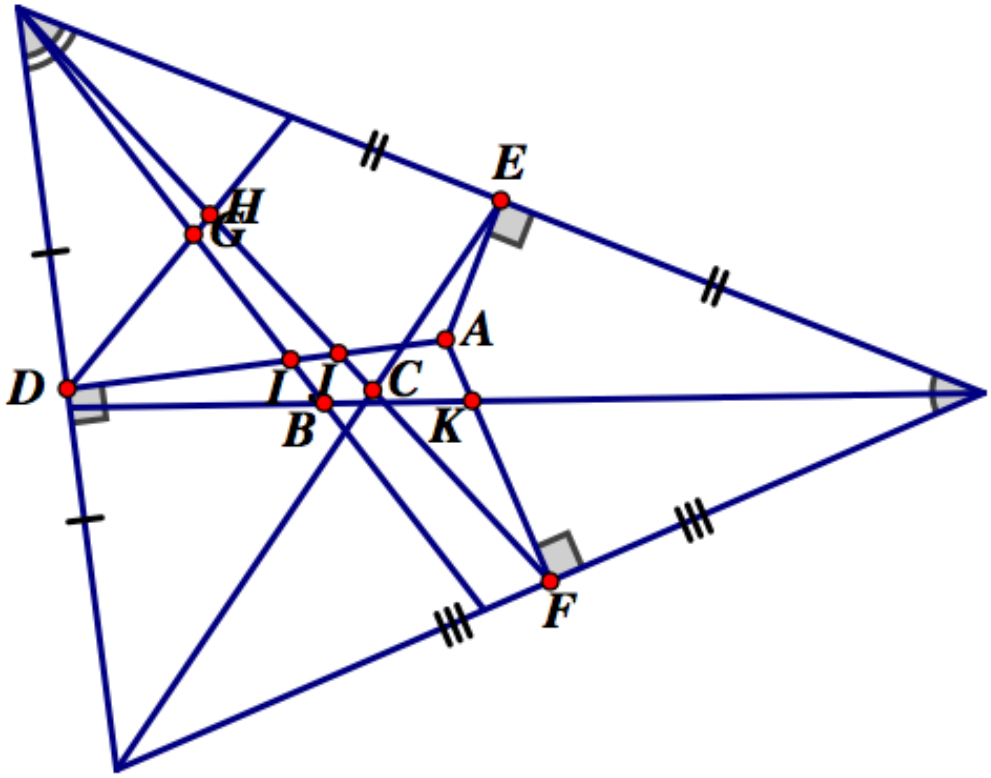


Day 43

. In the figure at the right, C is the centroid of $\triangle DEF$.
If $GF = 12x^2 + 6y$, which expression represents CF?



Day 43

TOD - hand in

What point represents the circumcenter, the incenter and the centriod?